Approximating integrals using interpolating polynomials

1. Approximate the integral of te^{-t} from $t = 1$ to $t = 2$ with $h = 0.1$ and $h = 0.05$ using the Reimann sum (left end-point), trapezoidal rule, Simpson's rule, and the 1/24th centered rule.

Answer: 0.3345007803222043 0.3296403365872935 0.3297526997680225 0.3297539506518482

0.3321550648612877 0.3297248429938323 0.3297530117960118 0.3297530899761732

2. The integral in Question 1 is approximately 0.3297530326330465. Indicate which formulas are $O(h)$, which are $O(h^2)$ and which are $O(h^4)$, and support your claims based on the results shown in Question 1.

Answer: The errors, to four significant digits, of the above six results are

–0.004748	0.0001127	0.0000003329	–0.0000009180
–0.002402	0.00002819	0.00000002084	–0.00000005734

You will note the first error drops by approximately one half, the second by one quarter, and the last two by one sixteenth.

3. An accurate sensor is reading speed at a rate of once every five seconds, and the reading is in meters per second. The readings are as follows:

0, 0, 0, 0.05, 0.40, 1.26, 2.73, 4.75, 7.15, 9.63, 11.89, 13.69, 14.90

What is a reasonable approximation of the total (cumulative) distance traveled in meters at each reading without using future information?

Answer: Using the four-point composite rule, and rounding afterward, we have

0, 0, 0, 0, 1, 5, 15, 33, 63, 105, 159, 223, 295

Thus, if initially slowly accelerating up to 14.90 m/s (or 53.64 km/h) over a period of approximately 50 seconds, the distance travelled is just shy of 300 meters. If the individual had been travelling accelerating uniformly up to 14.9 m/s for the entire 50 seconds, the distance travelled would be $14.9 \times 50 / 2 = 372.5$ meters, so given that we are accelerating up to 14.9 m/s, it is reasonable that we are travelling a slightly smaller distance.

4. Suppose you know that your system had occasional discontinuities of a signal that you would like to integrate. What strategies could you use to still get a reasonable estimate as to the total integral?

Answer: First, determine how quickly a signal not containing a discontinuity can change over a time interval. Suppose that this is some value Δ_{max}. Under the assumption that discontinuities are significantly greater, if $|y_{k+1} - y_k| > 2\,\Delta_{max}$, assume there was a discontinuity and thus, use the trapezoidal rule over that interval, as well as the next. With the next interval, use the 3-point integration rule, and with the next and all subsequent intervals until the next discontinuity, use the 4-point integration rule.